HEAT TRANSFER FROM A CYLINDER IN A SOUND FIELD AT GRASHOF NUMBERS APPROACHING ZERO

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Transfer processes between a solid body and a liquid or gaseous medium are significantly intensified in the presence of oscillatory relative motion between the body and the medium [1-4]. Considerations connected with the mechanism of the interaction between sound vibrations and transfer processes are limited to speculations about the effect of sound on thermal and diffusion boundary layers. Published work indicates that, in the present state of our knowledge of the mechanism of this process, it is impossible to make a theoretical analysis of transfer processes in a sound field [5]. In this paper we shall set up certain relations that determine the intensity of heat transfer in a sound field for limiting values of the dimensionless groups which characterize the process.

Notation

- x – longitudinal coordinate
- y – transverse coordinates
- t – time
- u – longitudinal component of velocity
- v – transverse component of velocity
- s – displacement amplitude of particle
- ω – angular frequency of vibrations
- B – vibration speed
- λ – wavelength
- R – radius of cylinder
- T – temperature
- $\tau = \omega\tau$ – dimensionless time
- ν – kinematic viscosity
- δ_2 – thickness of thermal boundary layer
- D – thermal diffusivity
- P – Prandtl number
- G – Grashof number
- N – Nusselt number based on diameter
- N_0 – Nusselt number based on radius
- ψ – stream function
- T – wall temperature
- η – dimensionless coordinate
- δ_1 – thickness of momentum boundary layer

$$\eta = y\sqrt{\omega/2\nu}, \quad U = u/B, \quad V = v/B, \quad X = x/R, \quad Y = y/R$$

Indices and symbols: ' – pulsating component of temperature and velocity, $_0$ – stationary component of temperature and velocity, erf – error function, < > – time average.

§1. In dimensionless form, the equations of the momentum boundary layer in the Oxy system of coordinates (Fig. 1) are

$$\frac{\partial U}{\partial \tau}+\frac{s}{R}V\frac{\partial U}{\partial Y}+\frac{s}{R}U\frac{\partial U}{\partial X}=\frac{\nu}{\omega R^2}\frac{\partial^2 U}{\partial Y^2}+\frac{s}{R}U^\circ\frac{\partial U^\circ}{\partial X}+\frac{\partial U^\circ}{\partial \tau} \qquad (1.1)$$

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0, \; U=0, \; V=0 \quad \text{at} \quad y=0; \; U=U^\circ=2\sin X\cos\tau \quad \text{at} \quad y=\infty . \qquad (1.2)$$

Equation (1.1) and all following derivations hold for

$$\frac{\nu}{\omega R^2}\ll 1, \qquad G \ll 1, \qquad \frac{\lambda}{2\pi R} > 1. \qquad (1.3)$$

Consider the case $s/R \ll 1$. Experimental studies [6-8] show that in this case there develop in the fluid stationary circulating currents – the secondary flow (Fig. 2).

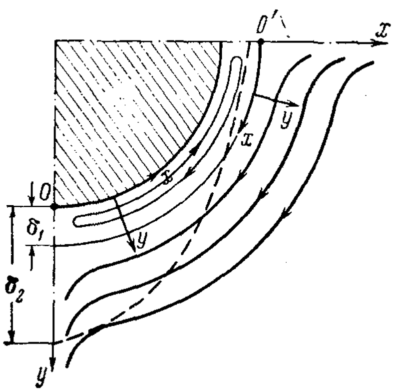

Fig. 1.

We shall assume that the rate of heat transfer is determined by the velocity of the secondary flow. Then the problem reduces to the determination of the temperature field in the thermal boundary layer, which develops from the generatrix of the cylinder that constitutes the stagnation line of the secondary flow. The velocity of the secondary flow was calculated by Schlichting [8], who solved the system of equations (1.1)-(1.2) by the method of successive approximations.

The longitudinal component of velocity is represented by Schlichting in the form $u = u_0 + u'$, where u_0 and u' are the stationary and pulsating components of velocity, respectively. The two components are

$$u_0 = 2\frac{B^2}{\omega R}\sin\frac{2x}{R}f(\eta) - \frac{3}{2}\frac{B^2}{\omega R}\sin\frac{2x}{R}$$
$$u' = 2B\sin\frac{x}{R}\left[\cos\omega t - e^{-\eta}\cos(\omega t-\eta)\right]. \qquad (1.4)$$

Here

$$f(\eta) = e^{-\eta}(1/2\eta + 2)\sin\eta + 1/2 e^{-\eta}(1-\eta)\cos\eta + 1/4 e^{-2\eta}.$$

The thickness of the momentum boundary layer is the same for the stationary and for the pulsating velocity components and is very small – equal to $\delta_1 = \sqrt{2\nu/\omega}$. At $y > \delta_1$, i.e., outside the momentum boundary layer, there exists a steady flow with longitudinal velocity

$$u_0 = -\frac{3}{2}\frac{B^2}{\omega R}\sin\frac{2x}{R}.$$

In the original system of coordinates Oxy (Fig. 1) the velocity of the secondary flow is negative. Rewriting the energy equation in the O'xy system (Fig. 1) with origin O' at the stagnation point of the secondary flow, we obtain

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = D\frac{\partial^2 T}{\partial y^2}$$
$$T = T_w \quad \text{at} \quad y = 0 \qquad (1.5)$$
$$T = 0 \quad \text{at} \quad y = \infty.$$

The longitudinal component of velocity is

$$u = \frac{3}{2}\frac{B^2}{\omega R}\sin\frac{2x}{R} - \frac{2B^2}{\omega R}\sin\frac{2x}{R} f(\eta) + 2B\cos\frac{x}{R}[\cos\omega t - e^{-\eta}\cos(\omega t - \eta)].$$

The transverse component of velocity is determined by (1.2). Let the temperature field be represented in the form

$$T = T_0 + T' \qquad (1.6)$$

where T_0 and T' are the stationary and time-dependent components, respectively. Substituting (1.6) in (1.5) and averaging (1.5) according to Reynolds' method, we obtain

$$u_0\frac{\partial T_0}{\partial x} + v_0\frac{\partial T_0}{\partial y} = D\frac{\partial^2 T_0}{\partial y^2} - \left\langle u'\frac{\partial T'}{\partial x}\right\rangle - \left\langle v'\frac{\partial T'}{\partial y}\right\rangle. \qquad (1.7)$$

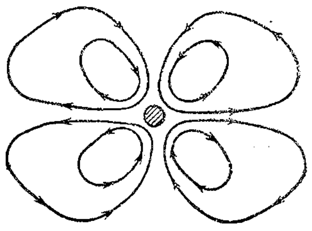

Fig. 2.

Here the last two terms on the right side represent the "pulsating heat conductivity".

Now we shall estimate the ratio of the thicknesses of the momentum and thermal boundary layers. Assuming, as usual,

$$\delta_2 = \left(\frac{R\nu}{u_0}\right)^n P^{-m},$$

we obtain for the case in question

$$\frac{\delta_1}{\delta_2} = \frac{(2\nu/\omega)^{1/2}}{(R\nu/u_0)^n P^{-m}}.$$

Taking for simplicity $n = 1/2$, we obtain

$$\frac{\delta_1}{\delta_2} \approx \frac{s}{R}P^m.$$

For $s/R \ll 1$ and $P \leq 1$ the thickness of the momentum boundary layer is much smaller than the thickness of the thermal boundary layer. For the range of Prandtl numbers characteristic of gases we may neglect the thermal resistance of the momentum boundary layer.

With this assumption, the velocity components inside the thermal boundary layer can be written in the form

$$u_0 = A\sin\frac{2x}{R}, \qquad v_0 = -\frac{2A}{R}y\cos\frac{2x}{R}\quad\left(A = \frac{3}{2}\frac{B^2}{\omega R}\right) \qquad (1.8)$$

$$u' = 2B\cos\frac{x}{R}\cos\omega t, \qquad v' = \frac{2B}{R}\sin\frac{x}{R}\left(y\cos\omega t - \frac{\delta_1}{2}\cos\omega t - \frac{\delta_1}{2}\sin\omega t\right) \qquad (1.9)$$

It is of interest to solve (1.7) neglecting the "pulsating thermal conductivity." We then have

$$u_0\frac{\partial T_0}{\partial x} + v_0\frac{\partial T_0}{\partial y} = D\frac{\partial^2 T_0}{\partial y^2} \qquad (1.10)$$

with the boundary conditions

$$T_0 = T_w \quad \text{at} \quad y = 0, \qquad T_0 = 0 \quad \text{at} \quad y = \infty.$$

In order to solve (1.10) we shall transform the equation from the rectangular coordinates x, y to von Mises' variables x, ψ. Keeping in mind that $u_0 = \partial\psi/\partial x$ and $v_0 = -\partial\psi/\partial y$, we obtain

$$\frac{\partial T_0}{\partial x} = AD \frac{\partial^2 T_0}{\partial \psi^2} \sin \frac{2x}{R}.$$

This equation can be reduced to the heat conduction equation

$$\frac{\partial T_0}{\partial \theta} = D \frac{\partial^2 T_0}{\partial \psi^2} \qquad \left(\theta = \int_0^x A \sin \frac{2x}{R}\, dx\right) \tag{1.11}$$

with the boundary conditions

$$T_0 = T_w \quad \text{at} \quad y = 0, \qquad T_0 = 0 \quad \text{at} \quad y = \infty, \qquad T_0 = 0 \quad \text{at} \quad \theta = 0,\ \psi = 0.$$

The solution of (1.11) is

$$T_0 = T_w \left(1 - \operatorname{erf} \frac{\psi}{2\sqrt{D\theta}}\right). \tag{1.12}$$

Using (1.12), we obtain the Nusselt number, based on the radius, for the cases under consideration

$$N_0 = \frac{\sqrt{6B}}{\sqrt{\pi\omega D}} \cos \frac{x}{R} \tag{1.13}$$

Time-averaged equation (1.7) contains the unknown value of the temperature pulsation in the thermal boundary layer.

Lighthill [9] has constructed a theory that makes it possible to calculate the value of the temperature pulsation from values of the velocity pulsation in a laminar boundary layer.

For $y > \delta_1$ the temperature pulsation is given by

$$T' = -\int u' \frac{\partial T_0}{\partial x} dt - \int v' \frac{\partial T_0}{\partial y} dt.$$

Using (1.9), we have

$$T' = -2B \cos \frac{x}{R} \frac{\sin \omega t}{\omega} \frac{\partial T_0}{\partial x} - \frac{2B}{R} \sin \frac{x}{R} \frac{\partial T_0}{\partial y} \left[\frac{y \sin \omega t}{\omega} - \frac{\delta_1}{2\omega} \sin \omega t + \frac{\delta_1}{2\omega} \cos \omega t\right]. \tag{1.14}$$

From (1.9) and (1.14) we can easily derive expressions for the "pulsating thermal conductivity" terms

$$\left\langle u' \frac{\partial T'}{\partial x} \right\rangle + \left\langle v' \frac{\partial T'}{\partial y} \right\rangle = -\frac{B^2 \delta_1}{\omega R^2} \cos \frac{2x}{R} \frac{\partial T_0}{\partial y}. \tag{1.15}$$

Substituting (1.8) and (1.15) in (1.7) and carrying out certain transformations, we obtain

$$\frac{\partial^2 T_0}{\partial \zeta^2} + k(X)\, \zeta \frac{\partial T_0}{\partial \zeta} = M(X) \frac{\partial T_0}{\partial X}$$

where

$$\zeta = 3y + \delta_1, \qquad M(X) = \frac{1}{6} \frac{B^2}{\omega R^2 D} \sin 2X, \qquad k(X) = \frac{1}{3} \frac{B^2}{\omega R^2 D} \cos 2X.$$

Using Shvets' transformation [10]

$$z = \zeta \left[e^{-c(X)} \int_0^X \frac{e^{c(X)}}{M(X)} dX\right]^{-1/2}, \qquad c(X) = 2 \int_0^X \frac{k(X)}{M(X)} dX$$

we obtain the equation

$$\frac{\partial^2 T_0}{\partial z^2} + \frac{1}{2} z \frac{\partial T_0}{\partial z} = 0,$$

the general solution of which is of the form

$$T_0 = C_1 + C_2 \operatorname{erf} (^1/_2 z).$$

Substituting the boundary conditions, we obtain

$$T_0 = T_w \left[1 - \operatorname{erf} \frac{(3y + \delta_1) B \cos X}{\sqrt{6\omega R^2 D}}\right] \left[1 - \operatorname{erf} \frac{B\delta_1 \cos X}{\sqrt{6\omega R^2 D}}\right]^{-1}$$

which after simple calculations leads to

$$N_0 = \frac{\sqrt{6}}{\sqrt{\pi}} \cos X \frac{B}{\sqrt{\omega D}} \frac{\exp[-{}^1/_3 P (s^2/R^2) \cos^2 X]}{1 - \operatorname{erf} {}^1/_3 \sqrt{3} (s/R) P^{1/2} \cos X}. \qquad (1.16)$$

Comparing (1.13) with (1.16), we see that pulsating heat transfer leads to the appearance of an additional multiplier, which is always somewhat greater than 1. For $s/R \ll 1$ and $P \leq 1$ this factor is practically equal to unity, and all heat transfer is due to the secondary acoustic flow. The rate of heat transfer is determined by (1.13), which, averaged over the cylinder, has the form

$$N = 1.76B^* \qquad \left(B^* = \frac{B}{\sqrt{\omega D}}\right). \qquad (1.17)$$

This equation connects the rate of heat transfer with the basic parameters of the sound field.

§2. During the experimental study of the heat transfer from a cylinder it was necessary to satisfy the following conditions:

(1) The effect of free convection must be negligibly small.
(2) The amplitude of particle displacement must be less than the characteristic dimension of the body, i.e., $s/R < 1$.
(3) The wavelength of the oscillations must be greater than the dimension of the body, i.e., $\lambda/2\pi R > 1$.

The first condition can be satisfied by appropriate choice of cylinder diameter and temperature difference, and the second and third conditions by appropriate choice of the parameters of the sound field.

In the experiments we investigated heat transfei from a wire in a standing wave field at the borderline between the sonic and ultrasonic ranges.

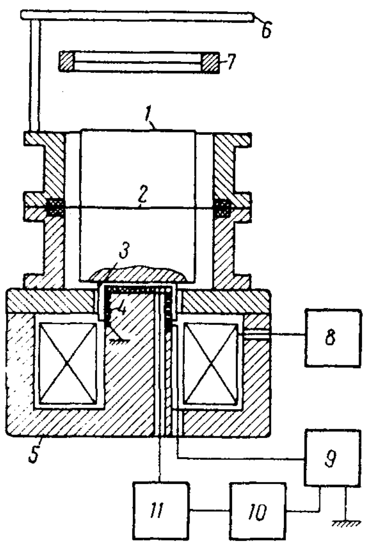

Fig. 3.

The high-frequency acoustic oscillations were obtained by means of an electrodynamic ultrasonic generator, which, unlike gas-jet sources, made it possible to obtain pure sinusoidal oscillations at a fixed frequency with sufficiently high sound intensity.

A diagram of the ultrasonic installation is shown in Fig. 3. Cylinder 1, the acoustic wave emitter, is held in the middle, where the oscillation amplitude is lowest, by a thin flange 2. A thin-walled ring 3, machined at the end of the cylinder, serves as the secondary winding of a high-frequency transformer, whose primary winding 4 is wound on the core of a dc magnet 5. The diagram also shows: 6 – reflector, 7 – calorimeter, 8 – magnetic-biasing rectifier, 9 – hf amplifier, 10 – phase converter, 11 – power supply. A current flowing in the primary winding of the high-frequency transformer induces a current in the shorted winding 3, which interacts with the fixed field of the magnet and generates a force that pushes or pulls the winding.

When the current frequency coincides with the natural frequency of vibration of the cylinder, the cylinder emits intensive acoustic waves. The operation of the emitter is stabilized by means of an electro-mechanical circuit, consisting of a feedback capacitor, an amplifying power-supply device, and a phase converter. The frequency and shape of the emitted oscillations is controlled by means of an ICh-6 frequency meter and a "Duoskop" oscillograph.

The intensity of the oscillations is measured by means of spherical barium titanate transducers (5 and 10 mm in diam.) on an AZ-2 accoustic probe, operating with a VZ-2A vacuum-tube voltmeter. The emitters used operated at 11.5 and 18 kilocycles. A flat metal screen is mounted above the upper end of the emitter in order to create a standing-wave field. Oscillation frequency at the antinodes of the standing waves was 160 dB (1 W/cm^2).

The calorimeter is made of platinum wire 195 μ in diam. and about 200 mm in length, mounted in the form of a flat coil on 50-μ constantan tension wires in a special holder, in such a way that the whole coil lies in a constant-intensity sound field. The measuring circuit is shown in Fig. 4, where 1 – battery bank, 2 – rheostat, 3 – standard resistance, 4 – R2/1 potentiometer, 5 – switch, 6 – wire calorimeter, 7 – measuring section, 8 – ammeter.

In order to eliminate the effect of heat conduction from the ends of the wire to its middle section, the measuring section 7 (Fig. 4) 10 mm in length is connected to the measuring instruments by means of 12μ tungsten wires, attached to the end points of the measuring section by spot welds.

The diameter and length of the measuring section were measured on an IZA-2 horizontal comparator with a MOV-1-15 screw-type ocular micrometer correct to within several microns. Temperature was determined from the temperature dependence of the resistivity of platinum [11] to within 1%. The temperature of the ambient air was measured with a copper-constantan thermocouple.

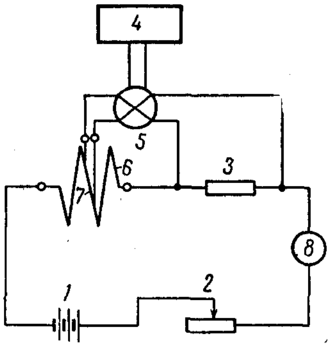

Fig. 4.

The resistance of the measuring section at 20°C was determined by calculation, as well as by measurement of the voltage drop at very low currents.

During the experiments the calorimeter was positioned in a plane with a fixed oscillation intensity above the emitter. Instrument readings were taken after the temperature of the measuring section reached a constant value. In order to satisfy condition (1), the experiments were carried out with temperature differences between 25 and 120°C.

The results of the preliminary measurements of free convection were practically identical with published data [12]. The analysis of the experimental data on heat transfer in a sound field indicates that all the results can be well represented in the form of a relation between the Nusselt number and the Peclet number based on the velocity of the secondary flow.

The results of the theoretical solution and of the experiments are shown in Fig. 5. The agreement can be regarded as satisfactory.

Thus it turns out that for $s/R < 1$ heat transfer grows linearly with increasing oscillation velocity in the sound wave, while at constant oscillation intensity an increase in sound frequency leads to a decrease in heat transfer.

Equation (1.17), which is corroborated by experiment, can be useful for estimating the effect of the parameters of a sound field on heat and mass transfer processes.

§3. When the amplitude of particle displacement is much greater than the dimension of the body, i.e., $s/R \gg 1$, the heat transfer process can apparently be regarded as quasi-stationary. Using, for example, McAdams' formula for heat transfer from a cylinder to a stationary stream [13]:

$$\frac{N}{P^{0.3}} = 0.35 + 0.56\left(\frac{ud}{\nu}\right)^{0.52}$$

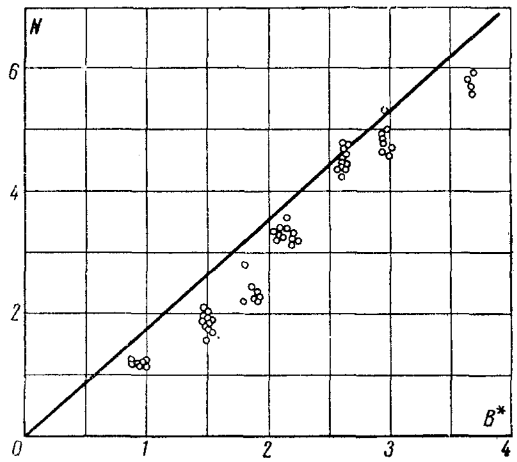

Fig. 5.

one can easily obtain the time-averaged Nusselt number for harmonic oscillations

$$\frac{N}{P^{0.3}} = 0.35 + 0.484\left(\frac{Bd}{\sqrt{2}\nu}\right)^{0.52}. \tag{3.1}$$

It is interesting that (3.1) exactly coincides with the experimental formula of Deaver, Penney and Jefferson [3].

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